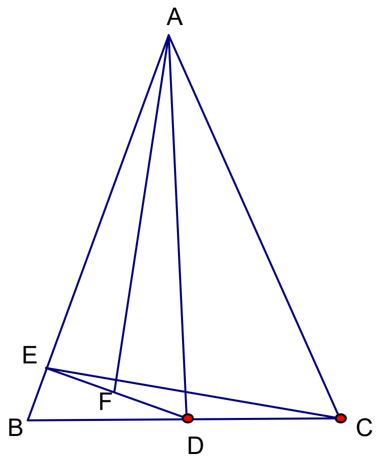
Geometry question
https://www.linkedin.com/groups/8313943/8313943-6411845590921076740
In triangle ABC, $AB = AC$ and D is the midpoint of side BC. Point E lies on side AB with $DE \perp AB$, and F be the midpoint of segment DE. Prove that $AF \perp EC$.



Solution by Arkady Alt , San Jose, California, USA.
WLOG we can assume that $\left\|\overrightarrow{AB}\right\| = \left\|\overrightarrow{AC}\right\| = 1$. Then $\overrightarrow{AD} = \frac{1}{2}\left(\overrightarrow{AB} + \overrightarrow{AC}\right)$,
$\overrightarrow{AD} = \cos\frac{A}{2}$, $\overrightarrow{AE} = \overrightarrow{AB}\cos^2\frac{A}{2} = \frac{1}{2}\overrightarrow{AB}(1+\cos A)$, $\overrightarrow{AF} = \frac{1}{2}\left(\overrightarrow{AE} + \overrightarrow{AD}\right) =$
$\frac{1}{2}\left(\frac{1}{2}\overrightarrow{AB}(1+\cos A) + \frac{1}{2}\left(\overrightarrow{AB} + \overrightarrow{AC}\right)\right) = \frac{1}{4}\left(\overrightarrow{AB}(2+\cos A) + \overrightarrow{AC}\right)$,
$\overrightarrow{EC} = \overrightarrow{AC} - \overrightarrow{AE} = \overrightarrow{AC} - \frac{1}{2}\overrightarrow{AB}(1+\cos A) = \frac{1}{2}\left(2\overrightarrow{AC} - \overrightarrow{AB}(1+\cos A)\right)$.
Hence and since $\overrightarrow{AC} \cdot \overrightarrow{AB} = \cos A$ we obtain
$8\,\overrightarrow{AF} \cdot \overrightarrow{EC} = \left(\overrightarrow{AC} + \overrightarrow{AB}(2+\cos A)\right) \cdot \left(2\overrightarrow{AC} - \overrightarrow{AB}(1+\cos A)\right) =$
$2\overrightarrow{AC} \cdot \overrightarrow{AC} + \overrightarrow{AB} \cdot \overrightarrow{AC}(3+\cos A) - \overrightarrow{AB} \cdot \overrightarrow{AB}(2+\cos A)(1+\cos A) =$
$2 + (3+\cos A)\cos A - (2+\cos A)(1+\cos A) = 0$